Old Math from Japan

Japanese children have always learned math through stories. These stories date back to the 17th and 18th centuries. Some of them are still used in contemporary Japan to teach elementary and junior high students. They are a wonderful way to introduce new math skills, and can be used in the context of story writing as well. (All stories are taken and adapted from Nihon no Sanpo, by Gisaku Nakamura, Kodansha Blue Backs, 1994.)

(1) The Arithmetic of Poppy seed

In one poppy seed there are nine worlds.
In one world there are nine seas.
In one sea there are nine mountains.
In one mountain there are nine valleys.
In one valley there are nine countries.
In one country there are nine provinces.
In one province there are nine villages.
In one village there are nine houses.
In one house there are nine children.
And on each child's tongue there are nine poppy seeds.
How many seas, mountains, valleys, countries, provinces, villages, houses, children and poppy seeds in the first poppy seed?

The answers are powers of nine. Note that if you add all the digits in each answer, they always come out as multiples of nine.

9^1 = 9 worlds		
9^2 = 81 seas	8+1=9	
9^3 = 729 mountains	7+2+9=18	1+8=9
9^4 = 6,561 valleys	6+5+6+1=18	1+8=9
9^5 = 59,049 countries	5+9+0+4+9=27	2+7=9
9^6 = 531,441 provinces	5+3+1+4+4+1=18	1+8=9
9^7 = 4,782,969 villages	4+7+8+2+9+6+9=45	4+5=9
9^8 = 43,046,721 houses	4+3+0+4+6+7+2+1=27	2+7=9
9^9 = 387,420,489 children	3+8+7+4+2+4+8+9=45	4+5=9
9^{10} = 3,486,784,401 seeds	3+4+8+6+7+8+4+4+1=45	4+5=9

(2) The Arithmetic of Travelers

Four friends traveled together
to a village six leagues away.
They had only three small horses.
Each wanted an equal share of ride.
One horse could only carry
one person at a time.
How did they work it out?

The total distance the three horses will walk is
$6 \times 3 = 18$ leagues
Therefore, each traveler is entitled to ride for
$18 \div 4 = 4.5$ leagues
Divide the six-league distance by four and name each section I, II, III, IV, respectively. Name the travelers A, B, C, D, and the horses a, b, c. The following chart shows how they worked it out.

Horse	I	II	III	IV
a	A	A	A	B
b	B	C	B	C
c	C	D	D	D

(3) The Arithmetic of Children

Yoshitsune, the famous *samurai* warrior, was traveling with a servant. A kind farmer invited them to his home for a meal. *Yoshitsune* saw many children playing in the backyard, and asked the farmer's wife, "How many children do you have?"

"Seven belong to my husband, and five belong to me. Together we have eight," was her answer.

"I don't understand. There should be twelve altogether," said the servant. *Yoshitsune*, however, understood and smiled.

How did this work?

For both parents it was their second marriage, and each had children from the previous marriages.

$7 + 5 - 8 = 4$ They had four children from this marriage.
$7 - 4 = 3$ The father had three from his first marriage.
$5 - 4 = 1$ The mother had one from hers.
$3 + 1 + 4 = 8$ So there are eight children in the family.

(4) The Arithmetic of Thieves Motoko’s Old Math from Japan

Some thieves were gathered underneath a bridge, trying to divide bolts of silk they stole from a merchant. Each one demanded his equal share. First they tried to take seven bolts each, but they were five bolts short. So each instead took six bolts, and there were three leftovers.

How many thieves were there, and how many bolts of silk?

If you add five bolts to the number of bolts they stole, each thief can take seven bolts. If, in that situation, each had decided to take six instead, the leftover would have been

3 + 5 = 8

This would have meant each gave up one bolt, so eight is the number of thieves. The number of the bolts of silk is

7 x 8 - 5 = 51 or 6 x 8 + 3 = 51

(5) Race Between a Horse and a Cow

A horse said to a cow, "I can run much faster than you."

"How fast can you run?" asked the cow.

"I can run thirty miles per hour, and I never get tired," bragged the horse, "How fast can you go?"

"I can only go five miles per hour. But I 'm better at math than you," said the cow.

"What do you mean?" asked the horse.

"You run thirty miles an hour, and I run five. If we go around a lake that's one hundred miles all around, starting at the same spot at the same time running in the same direction, how many hours will it take you to go all the way around and pass me again? Can you figure that out?"

The horse couldn't. Can you?

When the horse passes the cow, he would have run 100 miles more than the cow did. There difference in speed is

30 - 5 = 25.......... 25 miles per hour. So it will take the horse

100÷ 25 = 4 ....4 hours to pass the cow again.

(6) Meeting of Two Lovers

A woman lived in Kyoto, and her lover lived in Edo. They were one hundred and twenty-three leagues apart. They missed each other terribly, and decided to meet somewhere in between. The woman would walk twelve leagues a day, the man thirteen leagues. She left on the first day of the month. He left on the fifth. How many days did each have to travel before they were reunited?

The woman left four days before the man, and already had traveled:
12 x 4 = 48 leagues.

So when the man left Edo, the distance between them was
123 - 48 = 75 leagues.
This distance was covered by both of them, traveling each day
12 + 13 = 25 leagues.
75 ÷ 25 = 3 It took them three days to meet after the man left Edo.
The woman traveled for seven days, and the man three.

(7) Rabbits and Pheasants

Two brothers went hunting, and caught lots of rabbits and pheasants. They put them all in a big sack, and headed home,

"How many did we catch?" the younger brother asked.
"You count their heads. I'll count their feet," said the older brother.
The younger brother counted thirty-two, the older brother ninety-four.
How many rabbits and how many pheasants did they carry in that sack?

There were 32 creatures. If all of them were pheasants, there would have been 64 feet. But the actual number of feet was 94. The difference was due to the rabbits.
94 - 64 = 30 One rabbit means two more feet. Therefore,
30÷ 2 = 15 This is the number of rabbits.
32 - 15 = 17 This is the number of pheasants.

Or use variables: Rabbits = x; pheasants = y
$x + y = 32$
$4x + 2y = 94$

$x = 32 - y$
$4(32-y) + 2y = 128 - 4y + 2y = 128 - 2y = 94$
$y = (128 - 94) \div 2 = 17$

(8) The Girl Who Sold Flowers

Once there was a young girl who lived alone with her mother. They lived at the foot of a mountain north of Kyoto. Her mother was sick and had to stay in bed. So the girl worked hard to take care of her. In springtime the girl picked flowers in the mountain everyday and went to the city to sell them.

She picked four kinds of flowers: cherry, plum, peach, and apricot. Each day she sold three kinds, changing the combination to offer variety.

How many different combinations did she have?

To pick 3 kinds of flowers out of 4 is the same as to not pick 1 out of 4. Therefore the girl has the following 4 combinations.

- cherry, plum, peach (No apricot)
- plum, peach, apricot (No cherry)
- peach, apricot, cherry (No plum)
- apricot, cherry, plum (No peach)

(9) **Geometry for a Rice Merchant**

Once there was a rice merchant who had a young son. The boy was very smart, especially good at math. This pleased his father. One day the father showed the son a stack of large straw sacks filled with rice in front of their store. The sacks were full and shaped like barrels, and they were piled neatly in the shape of a trapezoid, the way people stack firewood. There were several rows with each row having one less sack than the row below it.

"There are eight sacks in the top row, and eighteen at the bottom," the father said, "Can you figure out how many sacks there are, without counting every single one?"

The boy did. Can you?

Draw the trapezoid as specified. Imagine another trapezoid just like the first one, turn it upside down, and attach it on one side of this trapezoid. Together they would make a parallelogram. The number of sacks in each row would be
8 + 18 = 26, because you are adding the top row and the bottom row.
As for the height of the pile, subtracting the top number (8) from the bottom number (18) gives you the number of rows sitting on top of the bottom row. So you must add one for the total height.
18 - 8 + 1 = 11
So the parallelogram would include
11 x 26 = 286 sacks of rice. The original trapezoid is half as much. Therefore,
286÷2 = 143 sacks.

(10) How Old Are You?

"Grandpa, how old are you?"
"How old do you think I am? Try and guess."
"Okay. If you divide your age by seven, what would the remainder be?"
"Two."
"If you divide it by five?"
"The remainder is one."
"And if you divide it by three?"
"The remainder is two."
"I got it! You are eighty-six!"
"That's very good. How did you guess?"

The smallest number divisible by 3 and 5 that can be divided by 7 with a remainder of 1 is 15.
The smallest number divisible by 3 and 7 that can be divided by 5 with a remainder of 1 is 21.
The smallest number divisible by 5 and 7 that can be divided by 3 with a remainder of 1 is 70. Therefore,
15 x 2 + 21 x 1 + 70 x 2 = 191 is the number that can be

- divided by 7 with a remainder of 2
- divided by 5 with a remainder of 1
- divided by 3 with a remainder of 2

Now the least common multiple of 7, 5, and 3 is
7 x 5 x 3 = 105 This is divisible by 7, 5, and 3. Therefore,
191 - 105 = 86 This is also a number that can be

- divided by 7 with a remainder of 2
- divided by 5 with a remainder of 1
- divided by 3 with a remainder of 2

To generalize this method:
$N = 15a + 21b + 70c$

- When N is divided by 7, a is the remainder.
- When N is divided by 5, b is the remainder.
- When N is divided by 3, c is the remainder.

You must check if N is larger than 105. If N is smaller than 105, that is the answer. If N is larger, than you must subtract 105 from it as many times as possible.